### Changing the gain at the cut-off frequency

Suppose that at some frequency *G* we have

.

Now consider a transformation, given by

, .

This has a few important properties. Every value inside the unit circle is mapped to a value inside the unit circle, those outside are mapped to values outside the unit circle, and those on the unit circle stay on the unit circle. So we can view this transfer function as preserving stability. Furthermore, it maps *z*=1 to 1 and *z*=-1 to -1.

But because of how a was defined, it has another important property,

.

That is, *F* maps this frequency G to /2. Hence, the square magnitude of *H*(*F*(*z*)) at /2 is the same as the square magnitude of *H*(*z*) at G.

### Shifting the cut-off frequency

So far, our cut-off frequency has been set to **/2. We would like to be able to set the cut-off frequency to any value between 0 and **. This would allow us to change the location that provides the transition between where frequencies are passed and blocked.

Consider another transformation, given by

,

This transfer function will map some new cut-off frequency *c* to the cut-off frequency of our prototype filter, **/2.

,

### Creating a shelving filter

We can construct a low shelving filter by transforming our prototype filter, such that the square magnitude response is transformed from *H*2 to (*G*2-1)*H*2­+1. This transformation changes the extreme square magnitudes 0 and 1 of a low pass design to 1 and *G*2.

Then the first order section of the low shelving filter becomes,

So

,

So b=0

Recall that the poles will push up the magnitude response for nearby frequencies, and the zeros will pull it down. For the low shelving filter, we want to keep the poles on the imaginary axis, giving a sharp cut-off frequency. But now we shift the zeros so that, for each first order section *H*(*z*=1)=*g* and *H*(*z*=-1)=1. That is achieved with.

So

,

,

### Inverting the magnitude response

For a high pass filter, we want |***z*=1)|2=0, |*z*=-1)|2=1 and |*z*=*c*)|2=G2. So we simply apply the transformation *F* (*z*)=-*z.*

### Simple low pass to band pass transformation

Now we want to create a band pass filter from our simple filter with centre frequency /2. We would like to transform the frequency range 0 to  to the frequency range – to If this transformation is applied to the input before a low pass filter is applied, then our low pass filter becomes a band pass filter. To do this, consider a transfer function *F*(*z*) with the following constraints;

.

*F* will move the lower and upper cut-off frequencies to ±/2, where our prototype low pass filter has its cut-off frequency, and it will move the centre frequency to 0, where our prototype low pass filter has gain equal to 1.

We can solve Eq. to arrive at

.

This transfer function has second order polynomials in the numerator and the denominator. So our first order section has now become a second order section.



Figure.1. Pole zero plot (top) and square magnitude response for a fourth order prototype low pass filter.



Figure 2. Shifting the centre frequency of a low pass filter.

Normalised frequency 

|H|2

0



/2

Normalised frequency 

|H|2

0



c

1

G2



1

1/2

/2



Figure 3. Shelving filter transformation.





Figure 4. Pole zero plot (top) and square magnitude response for a fourth order prototype low shelving filter (bottom). Compared to our prototype filter, it moves the zeros towards the pole positions on the imaginary axis.



Figure 5. Reversing the z domain to turn a low pass into a high pass filter.



Figure 6. Transforming a low pass filter into a band pass filter.



Figure.7. Pole zero plot for a 1st order (top left) and 4th order (top right) low pass filter with center frequency c=/4. On bottom, square magnitude response for the first order (solid line) and fourth order (dashdot line) filters.



Figure.8. Pole zero plot for a 1st order (top left) and 4th order (top right) high pass filter with center frequency c=/4. On bottom, square magnitude response for the first order (dashdot line) and fourth order (solid line) filters.



Figure.9. Pole zero plot for a 1st order (top left) and 4th order (top right) low shelving filter with center frequency c=/4 and gain at center frequency *G*=2. On bottom, square magnitude response for the first order (dashdot line) and fourth order (solid line) filters.



Figure.10. Pole zero plot for a 1st order (top left) and 4th order (top right) high shelving filter with c=/4 and *G*=2. On bottom, square magnitude response for the first order (dashdot line) and fourth order (solid line) filters.



Figure.11. Pole zero plot for a 2nd order (top left) and 8th order (top right) band pass filter with c=/4 and *B*=/8. On bottom, square magnitude response for the first order (dashdot line) and fourth order (solid line) filters.



Figure.12. Pole zero plot for a 2nd order (top left) and 8th order (top right) band stop filter with c=/4 and *B*=/8. On bottom, square magnitude response for the first order (dashdot line) and fourth order (solid line) filters.



Figure.13. Pole zero plot for a 2nd order (top left) and 8th order (top right) peaking filter with c=/4, *B*=/8 and *G*=2. On bottom, square magnitude response for the first order (dashdot line) and fourth order (solid line) filters.